# Yo-yo

Aim: To show the movement of a yo-yo and the force it excerts on the string that is holding it.

Subjects: 1Q20 (Rotational Energy)

Diagram:

Equipment:

- Yo-yo.
- Force sensor.
- Data-acquisition system.
- Beamer to project monitor image.

# Yo-yo

Presentation: We all know the yo-yo: Two circular discs with a common shaft and a string several times wrapped around it. Hold the end of the string stationary and release the yo-yo. The string unwinds as the yo-yo drops and rotates with increasing speed. When the unwrapping is completed, the yo-yo climbs again, comes to a stop and starts over again. etc.
Suspending the yo-yo to a force sensor, a registration of the tension in the string is made (red graph in Figure1). When, finally, the yo-yo has come to rest, such a registration is repeated (green line in Figure1).

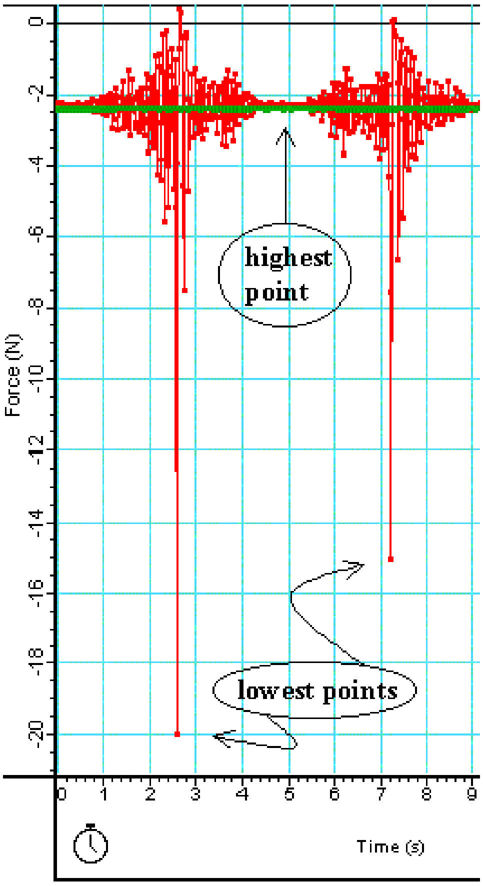


Figure 1

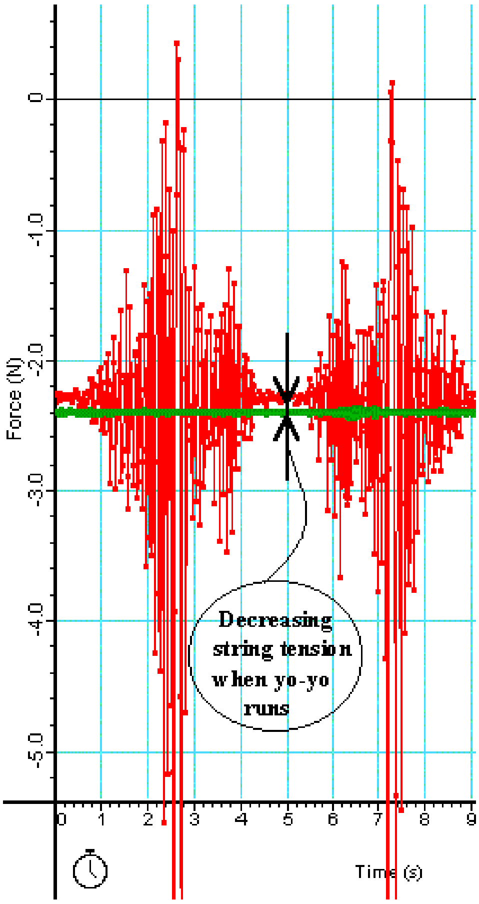


Figure 2

When studying these graphs, the jerk at the turning point is clearly observed. (Also a strong vibration.) See that the jerk at the turning point is much higher than the weight of the yo-yo.
Going from one jerk to the next, the highest position of the yo-yo is halfway between the two jerks. When a complete cycle is enlarged (see Figure2), it is clear that during the complete cycle the string tension is lower than the weight of the yo-yo.

# Yo-yo

Explanation: The yo-yo accelerates ($a$) due to a force

$ma=mg-F_s$ ($F_s$ being the string tension and $m$ the mass of the yo-yo.)

When there is no string, then $F_s$=0 and $a=g$ (free fall);

With a string, $a$ is always smaller than $g$:

$$a = g - \frac{F_s}{m}$$

When $F_s$ is just a little smaller than $mg$, then $a$ will be very small.

The angular acceleration ($\alpha$) of the roll during its fall can be found from $\alpha = \frac{\tau}{I}$, where the net torque ($\tau$) is given by $\tau=mgr$.

The acceleration of the center of mass ($a$) is related to the angular acceleration of the yo-yo by $a=\alpha r$, so the yo-yo accelerates downward by $a = \frac{mgr^2}{I}$.

Our yo-yo is a simple double disc, so $I_{CM} = \frac{1}{2}mR^2$. It rolls at the circumference of the shaft (radius $r$), that's why $I = \frac{1}{2}mR^2 + mr^2$, and we find for the acceleration:

$$a = \frac{2g}{2 + \frac{R^2}{r^2}}.$$

Because $R>>r$, $a<<g$.

With our yo-yo we have $R$=150mm and $r$=12mm, so $a$=0.012$g$.

Also the string tension can be calculated now: $F_s = mg - ma$, so: $F_s = mg - 0.012mg$, showing that the string tension is just a little lower than the weight of the yo-yo.

Remarks:

- A worthwhile observation is, that when the string is unwrapped completely and the yo-yo starts climbing again, that the yo-yo's translational velocity changes its direction ($-v$ to $+v$) but keeps its rotation in the same direction. In other words: its momentum changes direction ($-mv$ to $+mv$), but there is no change in angular momentum. The large change of momentum ($2mv$) at this point of the yo-yo's movement explains the jerk (the change of momentum takes place in a very short time).
- See also the demonstration "Maxwheel" in this database in order to link the measured string tension to the acceleration of the yo-yo. Usually we show these two demonstrations together.

Sources:


- Borghouts, A.N., Inleiding in de Mechanica, pag. 186-187
- Roest, R., Inleiding Mechanica, pag. 183-185
- Young, H.D. and Freeman, R.A., University Physics, pag. 303