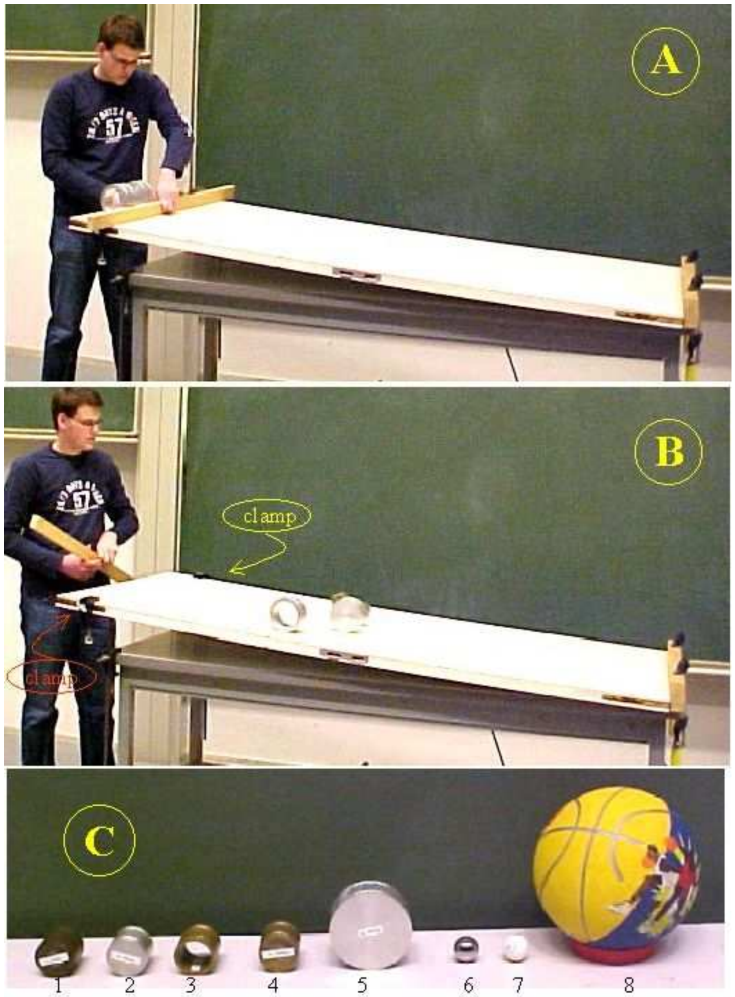
Rolling downhill

Aim: To show, qualitatively, the influence of the moment of inertia in rolling downhill.

Subjects: 1Q10 (Momentum of Inertia)
1Q20 (Rotational Energy)

Diagram:



Equipment:

- Inclined ramp (l=2m; we use a flat door).
- rolling objects
  1. m_1 cilinder of wood with lead in the center (ϕ60mm; m=424g); C=0,21
  2. m_2 solid aluminum cilinder (ϕ60mm; m=424g) C=0,5
  3. m_3 hollow brass cilinder (ϕ60mm, d=5mm; m=424g); C=0,83
  4. m_4 solid brass cilinder (ϕ60mm; m=1.3kg); C=0,5
  5. m_5 solid aluminum cilinder (ϕ120mm; m=1.7kg); C=0,5
  6. m_6 steel ball (ϕ40mm; m=230g); C=0,4
  7. m_7 ping pong ball (ϕ37.5mm; m=2gram); C=0,67
  8. m_8 basketball; C=0.67
- Overhead sheet with list of moments of inertia of rolling objects.
- Balance with large display.

Rolling downhill

Presentation: <u>Preparation.</u>

The ramp has to be adjusted horizontally in its cross direction, using an air-level. The two clamps (see Diagram B) are placed in such a way that in starting, the shelf, while pressed against these clamps, is nicely perpendicular to the long side of the ramp. A heavy wooden beam keeps the ramp at the end of the table. This beam also stops the rolling objects.

<u>Presentation.</u>

Different races are presented to the students. Before each race, the mass of the racing objects is determined by placing each object on the balance. Then students are asked to predict the result of that race: Is there a winner/loser? When the answer is yes, which object will be the winner/loser?

- <u>Race 1:</u>
  m_1, m_2 and m_3 race down the ramp together, starting at the same time. m_1 is the winner, m_3 the looser. (Most students predict the right answer. Evidently the distribution of mass is important.)
- <u>Race 2:</u>
  m_2 and m_4 race down the ramp together. There is no winner. (Most students predict the wrong answer. Evidently mass is not important in this demonstration. It makes me remember Galileo dropping objects from the tower of Pisa, in which experiment mass has also no influence on the downward acceleration.)
- <u>Race 3:</u>
  m_2, m_4 and m_5 race down the ramp together. They arrive together; there is no winner. (Many students don't dare to predict. Evidently radius *R* is of no importance in this experiment.)

Now an overheadsheet is presented to the students that shows a table of the expressions of moments of inertia of the various rolling objects. When mass *m* and radius *R* are of no importance in these downhill races then the difference will be found in the factor ahead of mR^2 in the expressions of the moment of inertia. After this observation the next races will be predicted right by (almost) all students.

- <u>Race 4:</u>
  m_2, m_6 and m_7. m_6 is the winner, m_7 the loser.

Figure 1 shows a summary.

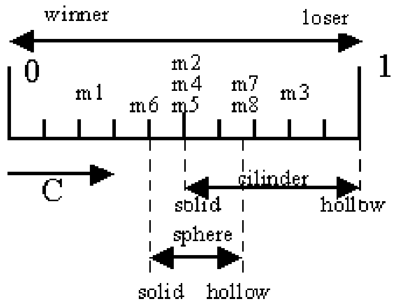


Figure 1

Explanation: Rolling down means translational acceleration plus rotational acceleration. The more

Rolling downhill

energy is needed for rotational acceleration, the less energy is left for translational acceleration.
Conservation of energy tells us: $1/2mv_c^2+1/2I\omega^2+mgh=constant.$

By $v_c=\omega R$, $h=s\sin\gamma$ and differentiating, we find $a_c = \dfrac{g\sin\gamma}{1+\dfrac{I_c}{mR^2}}$

The moment of inertia of objects with circular symmetry can be written as: $I=CmR^2$, where C is a constant. From tables we know (see Figure1):

- solid sphere, C=2/5
- solid cilinder, C=1/2
- hollow sphere, C=2/3
- hollow cilinder, C=1
- cilinder with thickness (R_2-R_1), $C=\frac{1}{2}\left(1+\left(\frac{R_1}{R_2}\right)^2\right)$

Using C in the expression above, we find: $a_c = \dfrac{g\sin\gamma}{1+C}$

The larger C, the smaller a_c. Also note that a_c does not depend on m or R!

Remarks:

- To have a fair race you need to have a good starting procedure. We get the best start by using a shelf, blocking the objects in the starting position. Starting is done by very quickly moving the shelf away into the direction of the descending ramp.
- When racing the basketball and ping pong ball (Race 5) there is no winner. Makes sure that in starting, the basketball is not ahead of the ping pong ball.

Sources:

- Jewett Jr., John W., Physics Begins With Another M... : Mysteries, Magic, Myth, and Modern Physics, pag. 113-114
- Young, H.D. and Freeman, R.A., University Physics, pag. 304
- Giancoli, D.G., Physics for scientists and engineers with modern physics, pag. 263