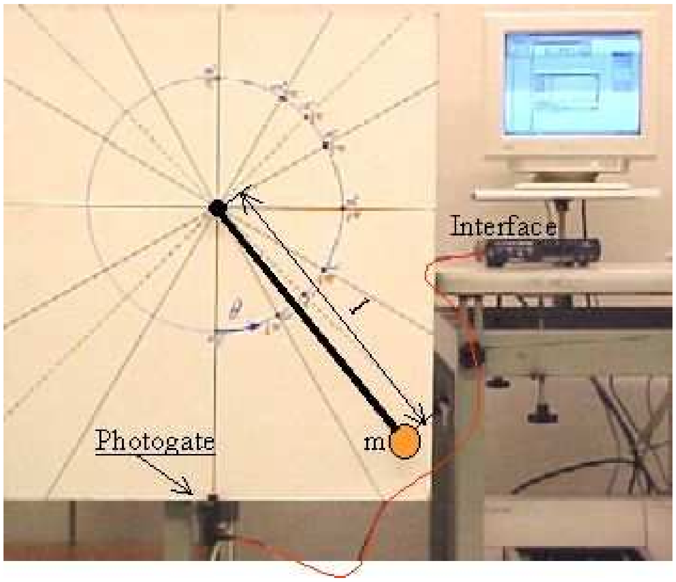
Mathematical pendulum (2)
Large angle

Aim: To show that the period of motion of a simple pendulum depends on the angle the pendulum makes with the vertical.

Subjects: 3A10 (Pendula)

Diagram:



Equipment:

- Pendulum; brass bob attached to a threaded rod (l=50cm) and connected to a support with ballbearing.
- Large cardboard with the principal angles of deflection indicated on it (see Diagram).
- Photogate.
- Computerinterface.
- Computer with data-acquisition system. (we use PASCO Science Workshop)

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Presentation: The photogate is placed just offset the rest-position of the pendulum. The data-acquisition system is set up in such a way that a graph of periodtimes can be presented. The data-acquisition is started, and by hand the pendulum is given a deflection of almost 180° and released. When θ has reached angles smaller than 90°, the data-acquisition is stopped. During the data-acquisition the students observe the graph displayed (see red line in Figure 1).

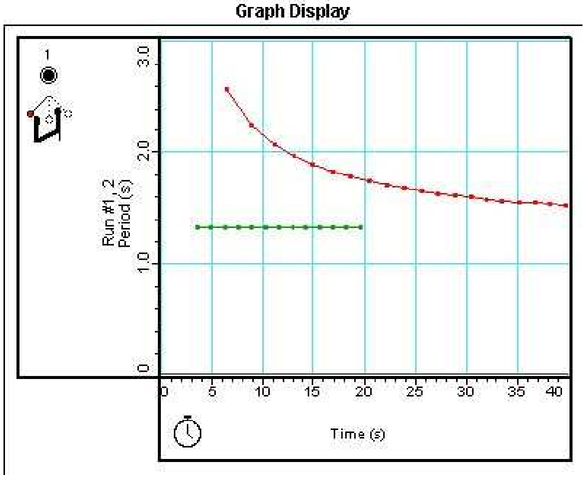

Figure 1

(This means that the expected range of the axes of the graph have to be prepared before the demonstration is started.)
A second run is made, giving the pendulum the smallest deflection possible. After about 10-20 registrations of T the data-acquisition is stopped. The complete graph can be observed and discussed now.

Explanation:

The equation that describes the motion of the mass m is given by $a_x = \frac{d^{2s}}{dt^2} = -g\sin\theta$ (x-direction along the tangent of the circle; see Figure2A). This is not a simple harmonic motion since *sinθ* is not proportional to *s*.

Only for small amplitude oscillations $\sin\theta \approx \theta = \frac{s}{l}$ and the equation of motion reduces to $\frac{d^2 s}{dt^2} = -\frac{g}{l}s$ This is the differential equation for simple harmonic motion. Then the period is given by $T = 2\pi\sqrt{\frac{l}{g}}$

For large amplitudes we need $a_x = -g\sin\theta$ in stead of $a_x = -g\theta$. Since $\sin\theta < \theta$, this means that a_x is smaller than the small-amplitude equation indicates: The mass will need more time than $T = 2\pi\sqrt{\frac{l}{g}}$ to reach its maximum deflection. In other words: *T* is

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larger than $2\pi\sqrt{\frac{l}{g}}$.

(For an exact solution to the equation of motion: see literature.)

Remarks:

- Also see the demonstration "Mathematical pendulum (1) - Simple harmonic motion" in this database. With that demonstration the effect on the acceleration *a* can be observed very well.
- When you observe the pendulum directly by eye it can be seen directly that the period of oscillation is larger at larger angles.
- The software is setup in such a way that the period is presented after the pendulum has passed three times through the photogate. Every next period is presented after every second passage (see Figure2B).

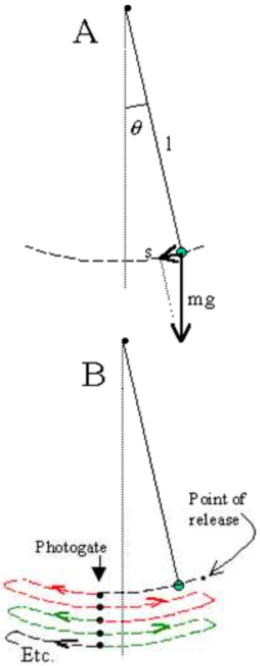

Figure 2

- Since the system measures the complete period the position of the photogate can be at any arbitrary point along the arc of motion.

Sources:

- Borghouts, A.N., Inleiding in de Mechanica, pag. 129-131
- Mansfield, M and O'Sullivan, C., Understanding physics, pag. 72-73
- Roest, R., Inleiding Mechanica, pag. 91-93